STEADY-STATE TURBULENT FLOW AND HEAT TRANSFER IN A DIFFUSER FORMED BY PARALLEL WALLS

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The drag and heat transfer in a diffuser formed by parallel walls (bladeless diffuser type) are considered for the case of fully developed turbulent flow. The theoretical solution of the problem is compared with the experimental data.

The investigation of the velocity fields and total-pressure losses in diffusers formed by parallel walls is of considerable practical importance, because diffusers of this type are used in centrifugal compressors.

There has also been a growing interest lately in cooled diffusers, which have applications in compressors designed for the compression of aggressive and dangerously explosive gases such as acetylene, chlorine, and others.

The theoretical analysis of heat transfer is possible after determination of the velocities only if the liquid can be treated as incompressible, or concurrently with the velocity determination if the influence of compressibility cannot be disregarded.

We begin by solving the first problem, i.e., we determine the velocity and pressure fields in a diffuser with a closed boundary layer.

We know from experiments [2] that the boundary layer in a bladeless diffuser closes rapidly, so that steady-state turbulent flow takes place in the main section of the diffuser.

The analogous problem for plane and axisymmetric channels has been solved earlier by Gurzhienko [5], Solodkin and Ginevskii [4], Sherstyuk [3], and others.

The special characteristics of the flow in the diffuser analyzed here (Fig. 1) require certain refinements of the Prandtl equation, which gives the relationship between the tangential stresses and velocity gradients.

The Reynolds equations, written for an axisymmetric steady flow of an incompressible liquid, assume the following form after certain terms have been discarded (in accordance with boundary-layer theory):

$$\bar{w}_r \frac{\partial \bar{w}_r}{\partial r} + \bar{w}_z \frac{\partial \bar{w}_r}{\partial z} - \frac{\bar{w}_\varphi^2}{r} = -\frac{1}{\rho} \cdot \frac{\partial \bar{p}}{\partial r} + \frac{1}{\rho} \cdot \frac{\partial}{\partial z}\left(\mu \frac{\partial \bar{w}_z}{\partial z} - \rho \overline{w_r' w_z'}\right), \tag{1}$$

$$\bar{w}_r \frac{\partial \bar{w}_\varphi}{\partial r} + \bar{w}_z \frac{\partial \bar{w}_\varphi}{\partial z} + \frac{\bar{w}_\varphi \bar{w}_r}{r} = \frac{1}{\rho} \cdot \frac{\partial}{\partial z}\left(\mu \frac{\partial \bar{w}_\varphi}{\partial z} - \rho \overline{w_\varphi' w_z'}\right), \tag{2}$$

$$\frac{\partial \bar{p}}{\partial z} = 0. \tag{3}$$

In Eqs. (1) and (2) $\tau_r = -\rho w_r' w_z'$; $\tau_\varphi = -\rho w_\varphi' w_z'$ are the Reynolds stresses.

The Reynolds stresses cannot be determined from the conventional Prandtl equation $\tau_r = \rho(l(\partial \bar{w}_r/\partial z))^2$; $\tau_\varphi = \rho(l(\partial w_\varphi/\partial z))^2$, because it was derived for two-dimensional flows.

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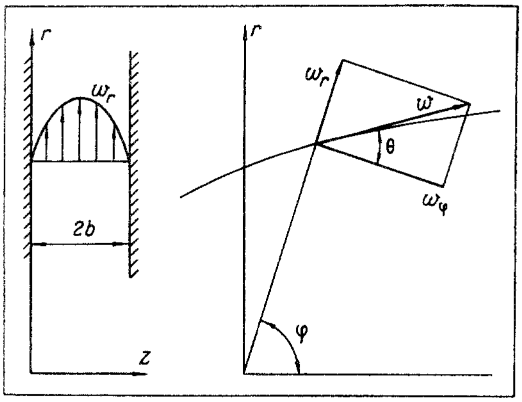

Fig. 1. Diffuser flow diagram.

The Prandtl equation has been generalized to the case of three-dimensional flows by Buleev [6]. However, Buleev's solution is exceedingly complex and is based on a number of empirical coefficients and hypotheses requiring careful experimental confirmation.

A simple way to generalize the Prandtl law to the three-dimensional case when the transverse component of the velocity is small in comparison with the longitudinal component has been indicated by A. N. Sherstyuk. By analogy with the Prandtl solution, the velocity fluctuations in the radial and circumferential directions can be written in the form

$$w_r' = l_1 \frac{\partial \bar{w}_r}{\partial z}; \quad w_\varphi' = l_1 \frac{\partial \bar{w}_\varphi}{\partial z}, \tag{4}$$

where l_1 is the distance traveled by one mole in the transverse direction until the loss of individuality.

The velocity fluctuation in the transverse direction must be proportional to the longitudinal velocity fluctuation:

$$w_z' = -c\sqrt{w_r'^2 + w_\varphi'^2}, \tag{5}$$

as is directly implied by the expression for the total tangential stress. Consequently, the tangential stresses τ_r and τ_φ are equal to, respectively,

$$\tau_r = -\rho\,\overline{w_r' w_z'} = \rho l^2 \frac{\partial \bar{w}_r}{\partial z}\sqrt{\left(\frac{\partial \bar{w}_r}{\partial z}\right)^2 + \left(\frac{\partial \bar{w}_\varphi}{\partial z}\right)^2}, \tag{6}$$

$$\tau_\varphi = -\rho\,\overline{w_\varphi' w_z'} = \rho l^2 \frac{\partial \bar{w}_\varphi}{\partial z}\sqrt{\left(\frac{\partial \bar{w}_r}{\partial z}\right)^2 + \left(\frac{\partial \bar{w}_\varphi}{\partial z}\right)^2}. \tag{7}$$

The mixing length l has the same sense as in the Prandtl equation. The value of l can be determined by means of the equations recommended by the authors in [7]. Hereinafter we consider only time-averaged variables, so that the average symbol can be dropped. Also, the problem is simplified to the extent that the laws governing the tangential stresses, $\tau_r = \tau_r(z)$ and $\tau_\varphi = \tau_\varphi(z)$, are approximated by polynomials, whose coefficients are determined from the boundary conditions:

1) $z = 0$; $\tau_r = \tau_{r0}$; $\partial\tau_r/\partial z = \partial p/\partial r$; $\tau_\varphi = \tau_{\varphi 0}$; $\partial\tau_\varphi/\partial z = \partial p/\partial z = 0$;

2) $z = b/2$; $\tau_r = \tau_\varphi = 0$.

Accordingly, we obtain

$$\bar{\tau}_r = \frac{\tau_r}{\tau_{r0}} = (1-\eta)\,[1 + (1 + A_{*r})\,\eta], \tag{8}$$

$$\bar{\tau}_\varphi = \frac{\tau_\varphi}{\tau_{\varphi 0}} = 1 - \eta^2. \tag{9}$$

Eliminating τ_r from (8) and (6) and τ_φ from (9) and (7) and taking the friction forces in the viscous substrate into account, we deduce two differential equations:

$$(1-\eta)[1 + (1 + A_{*r})\,\eta] = \frac{1}{\mathrm{Re}_{*r}} \cdot \frac{\partial v}{\partial \eta} + l^2 \frac{\partial v_r}{\partial \eta}\sqrt{\left(\frac{\partial v_r}{\partial \eta}\right)^2 + \left(\frac{\partial v_\varphi}{\partial \eta}\right)^2 \frac{\tau_{\varphi 0}}{\tau_{r0}}}, \tag{10}$$

$$1 - \eta^2 = \frac{1}{\mathrm{Re}_{*\varphi}} \cdot \frac{\partial v_\varphi}{\partial \eta} + \bar{l}^2 \frac{\partial v_\varphi}{\partial \eta}\sqrt{\frac{\tau_{r0}}{\tau_{\varphi 0}}\left(\frac{\partial v_r}{\partial \eta}\right)^2 + \left(\frac{\partial v_\varphi}{\partial \eta}\right)^2}$$

$$\left(v_\varphi = \frac{w_\varphi}{w_{\varphi *}};\ v_r = \frac{w_r}{w_{r*}};\ \mathrm{Re}_{*r} = \frac{w_{*r} b}{\nu};\ \mathrm{Re}_{*\varphi} = \frac{w_{*\varphi} b}{\nu}\right). \tag{11}$$

The set of equations (10)-(11) is readily solved by the method of iterations when the quantities Re_{*r} and $\mathrm{Re}_{*\varphi}$ are assumed to be known.

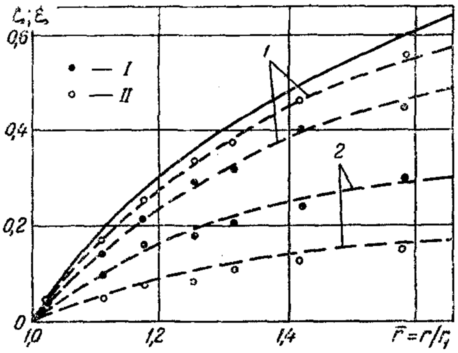

Fig. 2

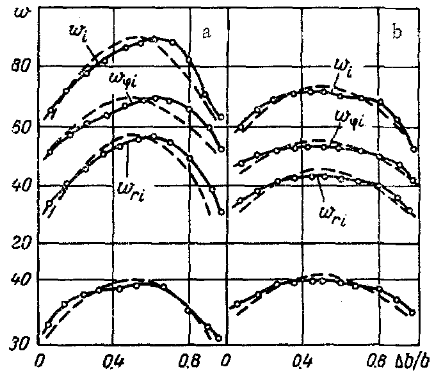

Fig. 3

Fig. 2. Diffuser characteristics. 1) $\xi = (p - p_1)/(\rho w_1^2/2)$; 2) $\zeta = (p_1^* - p^*)/(\rho w_1^2/2)$. Experimental data: I) $\theta_{1av} = 25.5°$, $Re_{\varphi 1} = 3.32 \cdot 10^4$, $\bar{b}_1 = 0.022$; II) $\theta_{1av} = 41°$, $Re_{\varphi 1} = 4.2 \cdot 10^4$, $\bar{b}_1 = 0.022$. Analytical data: solid curve) according to the relation $\xi = 1 - (1/\bar{r}^2)$; dashed curve) by the described procedure.

Fig. 3. Velocity fields (w, m/sec) and angles in the diffuser. The points represent the experimental data, $\bar{b} = 0.022$. a) $\theta_{av} = 37°$, $\bar{r} = 1.32$; b) $\theta_{av} = 39°$, $\bar{r} = 1.58$.

For the complete solution of the problem the relations thus derived must be augmented with two others, one of which relates the average circumferential projection w_φ^{av} of the velocity to the friction coefficient $c_{f\varphi}$ (stress $\tau_{0\varphi}$), while the second relates the pressure gradient (shape parameter A_{*r}) to the characteristics of the average flow.

The first relation is established by means of the theorem of angular momenta:

$$dM = G\frac{d}{dr}(rw_\varphi^{av})\,dr,$$

where

$$w_\varphi^{av} = \frac{1}{w_{r\,av}}\int_0^1 w_r w_\varphi d\eta; \quad w_{r\,av} = \int_0^1 w_r d\eta.$$

Finally we obtain

$$b\,\mathrm{tg}\,\theta_{av} - b_1\,\mathrm{tg}\,\theta_{av1} = \int_{r_1}^{r} c_{f\varphi}dr,$$
$$\mathrm{tg}\,\theta_{av} = \frac{w_{r\,av}}{w_\varphi^{av}}; \quad c_{f\varphi} = \frac{2\tau_{0\varphi}}{\rho(w_\varphi^{av})^2}. \tag{12}$$

The second relation is determined by means of the theorem of momenta:

$$b\frac{dp}{dr} = -\tau_{0r} + b\int_0^1 \rho w_\varphi^2\,d\eta - \frac{d}{dr}\left[br\int_0^1 \rho w_r^2 d\eta\right]$$

or

$$\frac{1 + A_{r*}}{v_{r\,av}^2} = k_r\,\mathrm{tg}\,\alpha + \frac{b}{r}\left[k_r + k_\varphi\,\mathrm{ctg}^2\,\theta_{av}\right], \tag{13}$$

where

$$k_r = \frac{1}{w_{r\,av}^2}\int_0^1 w_r^2 d\eta; \quad k_\varphi = \frac{1}{w_{\varphi\,av}^2}\int_0^1 w_\varphi^2\,d\eta;$$

$$w_{\varphi\,\mathrm{av}} = \int_0^1 w_\varphi d\eta; \ \operatorname{tg}\alpha = \frac{db}{dr}.$$

The procedure described here was used to calculate the velocity profiles, pressure increase, and total-pressure loss factor in the diffuser. The analytical results (dashed curves) are compared in Figs. 2 and 3 with the experimental data obtained on the All-Union Scientific-Research Institute for Cryogenic Machinery (VNIIkriogenmash) bladeless diffuser static test stand described in [8]. As the graphs indicate, the values of the local slope angles of the streamlines vary considerably within the cross section. This fact indicates a distinct three-dimensional flow pattern in the diffuser, resulting in an additional increase in the total-pressure losses and a decrease of the growth rate of the static pressure in the diffuser.

Knowledge of the velocity fields enables us to solve the heat-transfer problem for the diffuser.

The three-dimensional character of the flow prevents us from using the relation between the velocity fluctuations and temperature

$$\frac{T'}{T - T_0} = \frac{w'}{w}\,\mathrm{const}, \tag{14}$$

which was proposed in [9] and has been successfully used by the authors to calculate the heat transfer in a cylindrical pipe [7] and for flow past a plane wall [10].

A generalization of relation (14) to the case of a three-dimensional flow when the transverse velocity component w_z is small in comparison with the other components has also been given by A. N. Sherstyuk.

The transverse velocity fluctuation w'_z is assumed to be proportional to the longitudinal fluctuation: $w'_z = cw'$, but in the general case

$$w' \neq l_1 \frac{\partial w}{\partial z},$$

because the direction of the velocity vector w changes along z. We assume by analogy that

$$w' = \frac{l_1}{m} \cdot \frac{\partial w}{\partial z} \tag{15}$$

and determine the coefficient m. Inasmuch as $w = \sqrt{w_r^2 + w_\varphi^2}$, we therefore have

$$w = \sqrt{w_r^2 + w_\varphi^2}, \text{ то } \frac{\partial w}{\partial z} = \frac{1}{w}\left(w_r \frac{\partial w_r}{\partial z} + w_\varphi \frac{\partial w_\varphi}{\partial z}\right).$$

On the other hand, an expression for w'_z has been obtained previously [see Eq. (5)]. Therefore,

$$m = \frac{l_1}{w'} \cdot \frac{\partial w}{\partial z} = \frac{\sin\theta \dfrac{\partial w_r}{\partial z} + \cos\theta \dfrac{\partial w_\varphi}{\partial z}}{\sqrt{\left(\dfrac{\partial w_r}{\partial z}\right)^2 + \left(\dfrac{\partial w_\varphi}{\partial z}\right)^2}}. \tag{16}$$

According to Prandtl's theory

$$T' = l_{\mathrm{T}} \frac{\partial T}{\partial z}; \ w'_z = \frac{l}{m} \cdot \frac{\partial w}{\partial z} \ (l = l_1 c)$$

and, hence,

$$\frac{T'}{\dfrac{\partial T}{\partial z}} = \frac{l_{\mathrm{T}}}{l} m \frac{w'_z}{\dfrac{\partial w}{\partial z}}.$$

Transforming from the derivatives ∂T/∂z and ∂w/∂z to finite increments, we ultimately obtain the following relation between the velocity fluctuations and temperature:

$$\frac{T'}{T - T_0} = \mathrm{const}\, m \frac{w'}{w - w_0}. \tag{17}$$

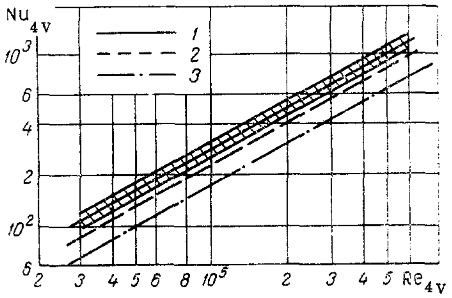

Fig. 4. Local heat transfer in a parallel-wall diffuser. The hatched strip represents the set of experimental data. 1) Averaged experimental dependence; 2) calculated by the described procedure; 3) calculated without regard for spatial spreading of the absolute velocity vector.

For two-dimensional flows m = 1 and relation (17) goes over to (14). It is easily shown that $m < 1$ always for three-dimensional flows. It is established below that this fact significantly increases the heat-transfer coefficient.

We write the heat-transfer law for turbulent flow with regard for the thermal conductivity of the liquid:

$$q = \lambda \frac{\partial T}{\partial z} - \rho c_p \overline{w_z' T'}. \tag{18}$$

The thermal mixing length is determined by relations given in [7].

Changing to dimensionless parameters, we rewrite the heat flux equation in the form

$$\bar{q} = \frac{q}{q_0} \approx 1 = \frac{1}{\mathrm{Re}_* \mathrm{Pr}} \cdot \frac{\partial \bar{t}}{\partial \beta} + \frac{v}{m} \varkappa^2 \mathrm{Pr}^{0.4} \beta^4 \frac{1}{4\beta_{v_1}^2} \cdot \frac{1}{\bar{t}} \left(\frac{\partial \bar{t}}{\partial \beta} \right)^2 \tag{19}$$

for the wall zone of the thermal layer, and in the form

$$\bar{q} = 1 - \frac{3}{2}\eta^2 + \frac{1}{2}\eta^3 = \frac{v}{m} \varkappa^2 \mathrm{Pr}^{0.4} \eta_1^2 \left(1 - \frac{\eta_1}{2}\right)^2 \frac{1}{\bar{t}} \left(\frac{\partial \bar{t}}{\partial \beta} \right)^2 \tag{20}$$

for the main part of the diffuser.

Using the solutions of the dynamical problem and invoking Eqs. (19) and (20), we can determine the relative temperature distribution across the channel. In particular, for the wall zone

$$\frac{\partial \bar{t}}{\partial \beta} = 2A_v \frac{\bar{t}}{v} \cdot \frac{m}{\beta^4} \left(\sqrt{1 + \frac{v\beta^4}{A_v m \bar{t}}} - 1 \right) \text{ for } \beta < \beta_{v1} \tag{21}$$

and

$$\frac{\partial \bar{t}}{\partial \beta} = 2A_t \frac{\bar{t}}{v} \cdot \frac{m}{\beta^2} \left(\sqrt{1 + \frac{v\beta^2}{A_t m \bar{t}}} - 1 \right) \text{ for } \beta_t > \beta > \beta_{v1}$$
$$\left(A_v = \frac{\beta_{v1}^2}{\varkappa^2 \mathrm{Pr}^{1.4}};\ A_t = \frac{1}{4\varkappa^2 \mathrm{Pr}^{1.4}} \right). \tag{22}$$

The value of β_{v1} is assumed to be the same as in a cylindrical pipe on a plane wall (see [7, 10]) with the total tangential stress at the diffuser wall taken into account.

In the main part of the channel the relative temperature distribution has the form

$$\bar{t}_\eta = \bar{t}_t + 2\sqrt{\bar{t}_t}\, f(\eta, A_{r*}, \theta_{av}) + f^2(\eta, A_{r*}, \theta_{av}), \tag{23}$$

where

$$f(\eta, A_{r*}, \theta_{av}) = \frac{1}{2\,\mathrm{Pr}^{0.2}} \int \frac{\sqrt{m}\sqrt{1 - \frac{3}{2}\eta^2 + \frac{1}{2}\eta^3}}{\varkappa \sqrt{v}\, \eta_1 \left(1 - \frac{\eta_1}{2}\right)} d\eta \tag{24}$$

is a function depending on the transverse coordinate, the pressure gradient, and the average flow angle.

Once the temperature distribution in the wall zone and in the main part of the channel has been determined, the number St can be found from the equation

$$\mathrm{St}=1\Bigg/\left\{\int_0^{\eta_v} v\bar{t}\,d\eta+\int_{\eta_v}^{\eta_t} v\bar{t}\,d\eta+\int_{\eta_t}^{1.0} v\left[\bar{t}_t+\frac{\sqrt{\bar{t}_t}\,f(\eta,\ \theta_{av}A_{*r})}{\varkappa \mathrm{Pr}^{0.2}}\cdot\frac{f^2(\eta,\ \theta_{av}A_{*r})}{4\varkappa^2\mathrm{Pr}^{0.4}}\right]\right\}. \tag{25}$$

The values of the numbers Nu_{4v} for local heat transfer in the diffuser are represented by the dashed line in Fig. 4 (for air, Pr = 0.7). Also shown in the same figure are local heat-transfer data obtained in an experimental investigation of an annular diffuser as described in [8] (hatched strip).

The graph clearly reveals the local heat-transfer increase induced by the three-dimensionality of the flow. Also shown as an example is the relation $Nu_{4v} = f(Re_{4v})$ for liquid flow and heat transfer in a plane slot.

A certain incongruity of the experimental and analytical results is clearly elicited by the presence in a real flow of detached localized zones, which add to the turbulence of the flow.

NOTATION

w	is the liquid flow velocity;
p	is the liquid pressure;
ν	is the kinematic viscosity coefficient;
p^*	is the total pressure;
ρ	is the density of the liquid;
τ	is the tangential stress;
$w_* = \sqrt{\tau_0/\rho}$	is the "frictional" velocity at the wall;
$v = w/w_*$	is the relative flow velocity;
$\eta = z/b$	is the relative coordinate;
$2b$	is the diffuser width;
$A_{r*} = (b/\rho w_{r*}^2)\cdot(dp/dr)$	is the relative pressure gradient in the radial direction;
$\bar{b} = b/r$	is the relative diffuser width;
α	is the angle formed by the diffuser walls in the meridian plane;
$\bar{r} = r/r_1$	is the relative diffuser radius;
θ	is the absolute velocity angle;
ξ	is the compression ratio;
ζ	is the total-pressure loss factor;
T	is the liquid temperature at a point;
λ	is the thermal conductivity;
l_1	is the "thermal" mixing length for one mole;
q	is the specific heat flux;
$\bar{t} = (T - T_0/T_{av} - T_0)\cdot(1/\mathrm{St}\, v_{av})$	is the relative temperature at a point;
St	is the Stanton number;
Pr	is the Prandtl number;
$\beta = w_* z/\nu$	is the relative coordinate in the wall layer;
$\varkappa$	is the turbulence constant;
Re	is the Reynolds number;
Nu	is the Nusselt number.

Subscripts

0	denotes the value of parameter at the wall;
v	denotes the boundary of viscous substrate;
t	denotes the boundary of transient region;
1	denotes the value of parameter at diffuser entry.

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